Footfall Induced Vibration

Arup Unified Method for Floors, Footbridges, Stairs and Other Structures

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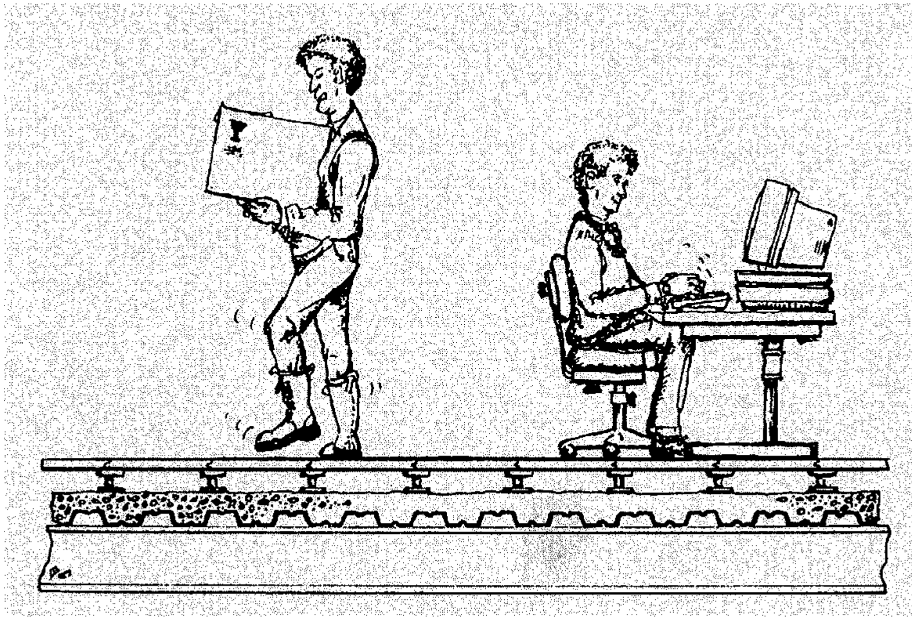
Outline

- Introduction to Footfall Induced Vibration
- Computing the Structural Response
- Project Examples
- References

Introduction to Footfall Induced Vibration

When is footfall induced vibration an issue?

- Low **Frequency** (Resonance)
- Low **Mass** (Acc=Force/Mass)
- Low **Damping**
- Large Dynamic Loads (Crowds)



The design problem

Loading:

- Amplitude
- Frequency
- Duration

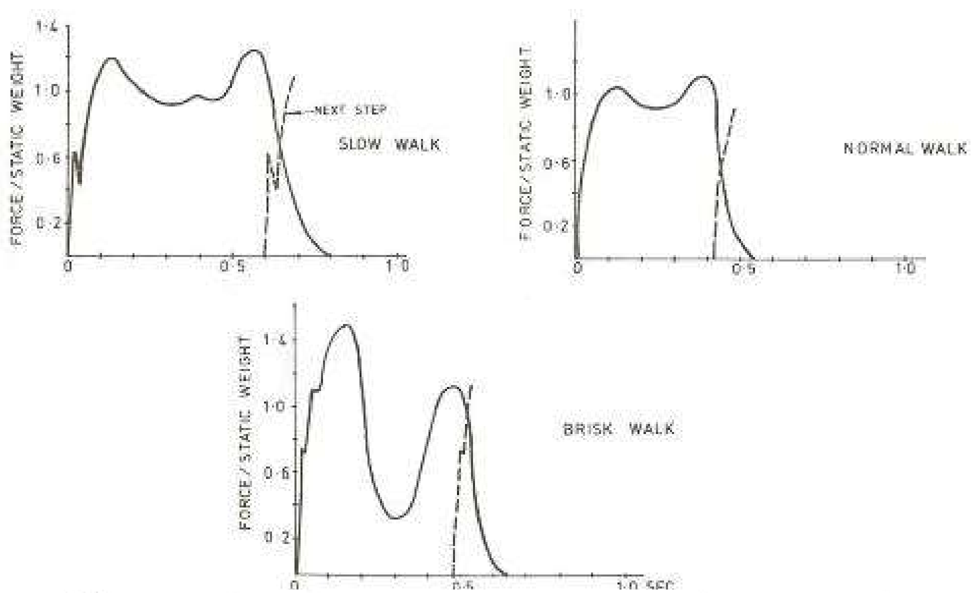
Modal (structural) properties:

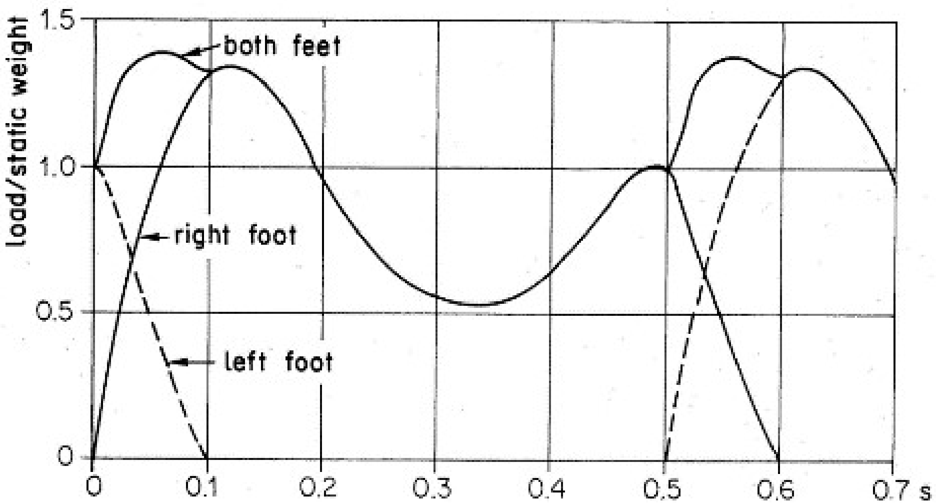
- Frequency
- Modal mass
- Mode shape
- Damping

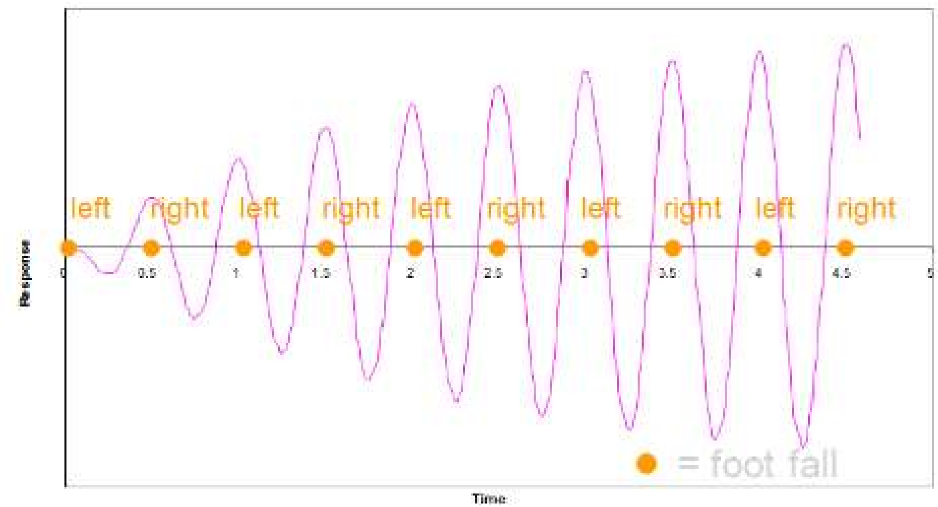
Response

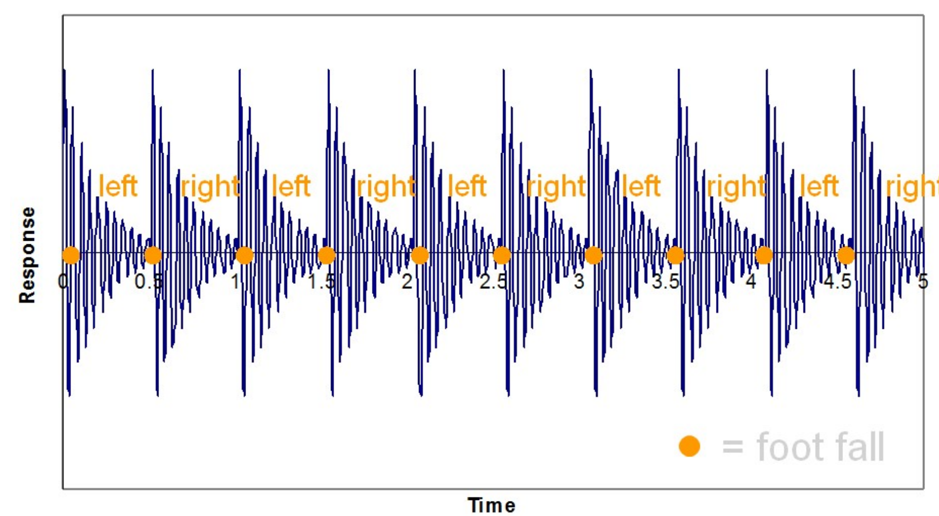
consequences,
acceptability, ?????

What does a footfall time history look like?



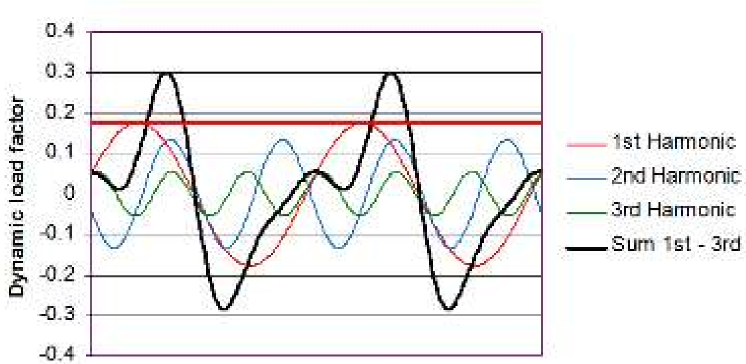


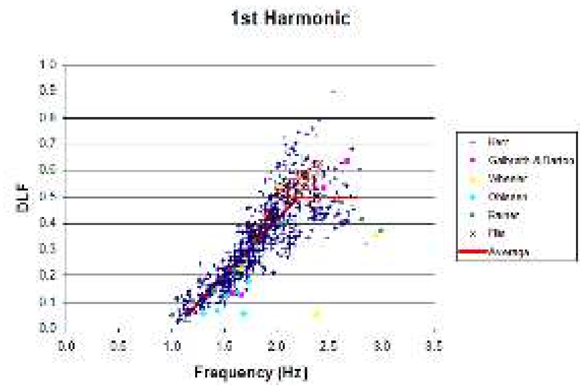


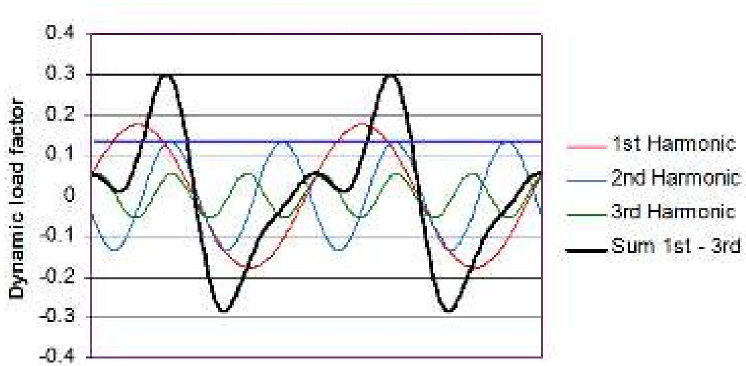


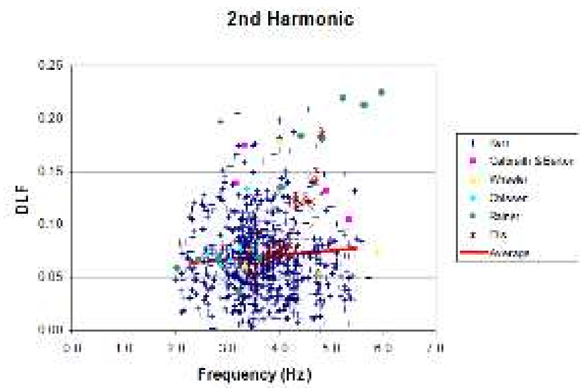
What kinds of excitation frequencies are possible?

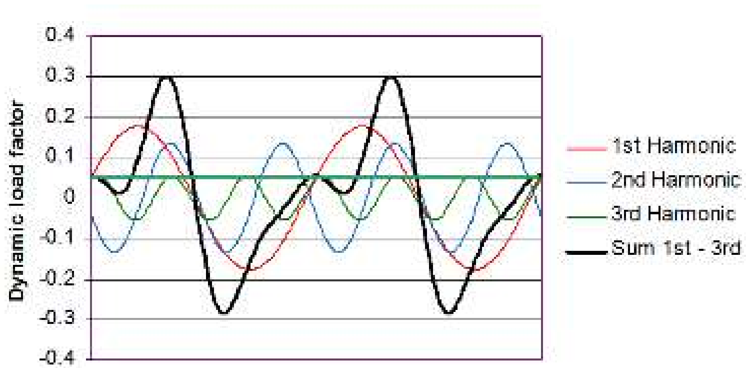
So how does this translate into a design load?

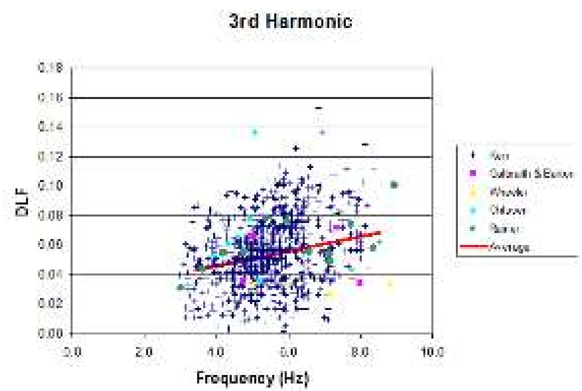










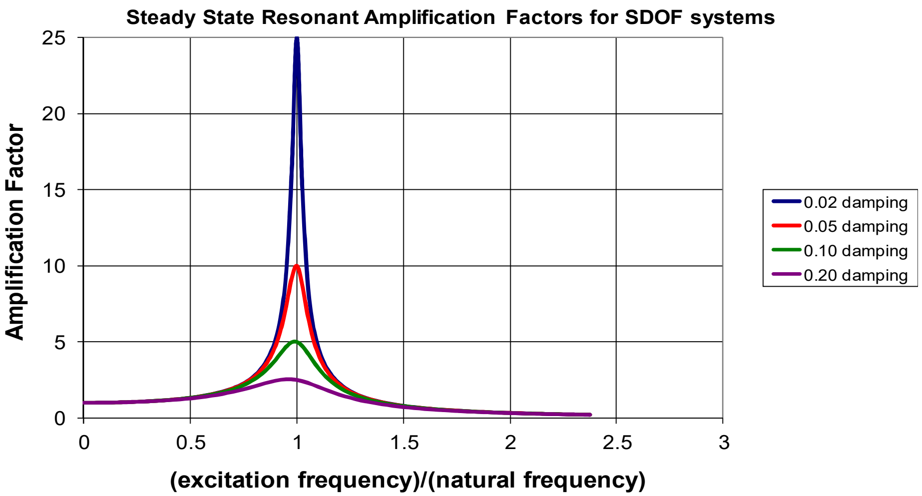


What is an appropriate amount of damping?

Damping :: Frequency :: Modal Mass :: Mode Shape

Be suspicious of >2%

Footbridges 0.5~1.5%
Stairs 0.5%
Floors 1~3%



Modelling Assumptions

Damping :: **Frequency** :: **Modal Mass** :: **Mode Shape**

…some tips (not exhaustive)…

Loads

- Use best estimate, not code values
- LL: ~0.5kPa typically realistic
- SDL: upper/lower bound sensitivity study

Model Extent

- Typically model single floor only
- Floorplate should capture all modes of interest

Boundary Conditions

- Fixed connections/supports unless true pin
- Façade vertical fixity may/may not be appropriate

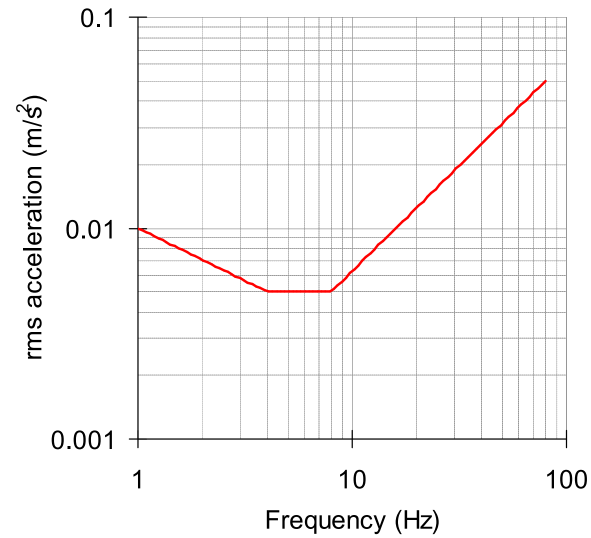
Member Modelling

- Orthotropic slab properties
- Composite beam (even if nominal studs)
- $E_{c,dynamic} \approx 38\text{GPa}$
- Subdivide 6+ elements per span

What is a good design criteria?

Situation	R	RMS Acc @ 5Hz
Low vibration	1	0.005 m/s^2
Residential (night)	1.4	0.007 m/s^2
Residential (day)	2-4	0.01~0.02 m/s^2
Office (high grade)	4	0.02 m/s^2
Office (normal)	8	0.04 m/s^2
Footbridge (heavy)	24	0.12 m/s^2
Footbridge (inside)	32	0.16 m/s^2
Footbridge (outside)	64	0.32 m/s^2
Stair (high use)	24	0.12 m/s^2
Stair (light use)	32	0.16 m/s^2
Stair (very light use)	64	0.32 m/s^2

Approximate threshold of human perception to vertical vibration

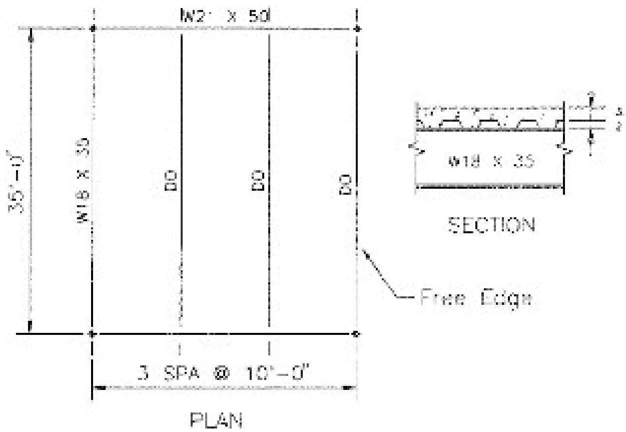


$$R = Acc_{RMS} / Acc_{RMS}$$

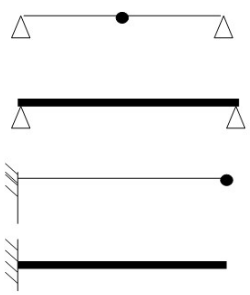
$$Acc_{RMS} = \sqrt{\frac{\sum_t^T Acc_t^2}{T}} = \frac{Acc_{Peak,Harmonic}}{\sqrt{2}}$$

Computing the Structural Response

Simplified vs Modal vs Time History Methods



Typically $f = \frac{18}{\sqrt{y}}$ for most structures Where: f is in cycles per second
y is the static deflection in mm



Simply supported Mass concentrated in centre	$f = \frac{15.8}{\sqrt{y}}$
Simply Supported Mass and stiffness distributed	$f = \frac{18}{\sqrt{y}}$
Cantilever Mass concentrated at end	$f = \frac{15.8}{\sqrt{y}}$
Cantilever Mass and stiffness distributed	$f = \frac{19.7}{\sqrt{y}}$

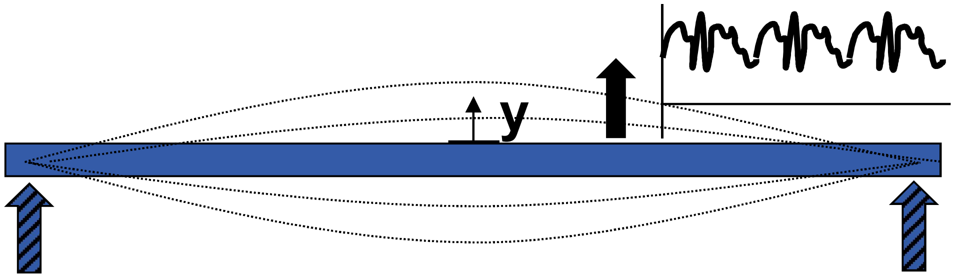
VS

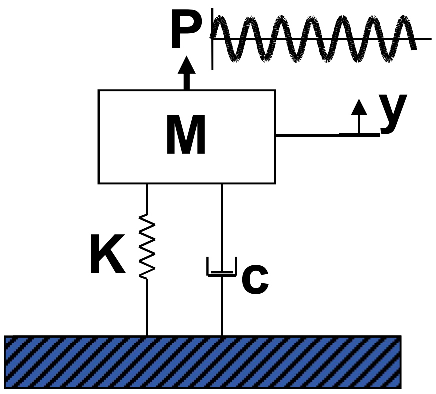
Modal harmonic response method

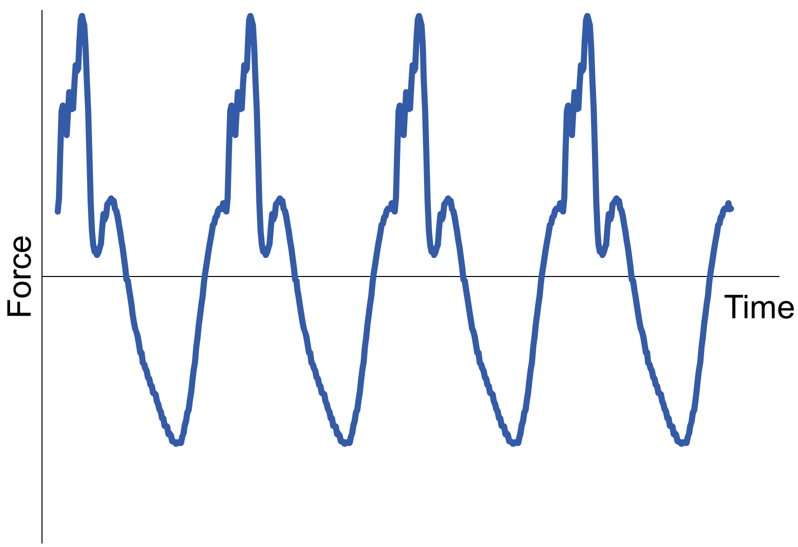
VS

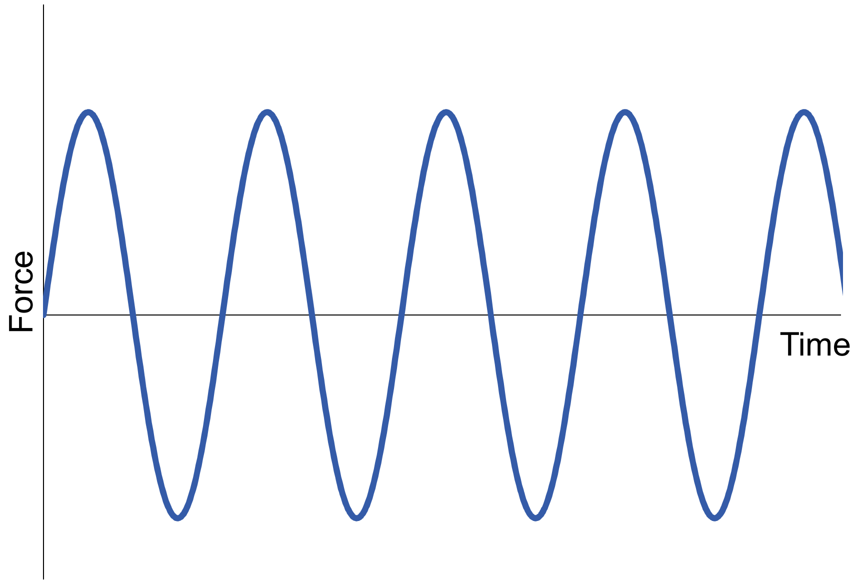
Explicit time history method

Analytical Model









Resonant Response

SDOF Harmonic Steady State Response

$$m\ddot{y}(t)+c\dot{y}(t)+ky(t)=P(t)=P\cdot e^{i\omega t}$$

$$FRF_{disp}=\frac{1}{k-m\omega^2+ic\omega}=\frac{1/k}{\left(1-\left(\frac{f_h}{f_m}\right)^2\right)+i2\xi\frac{f_h}{f_m}}$$

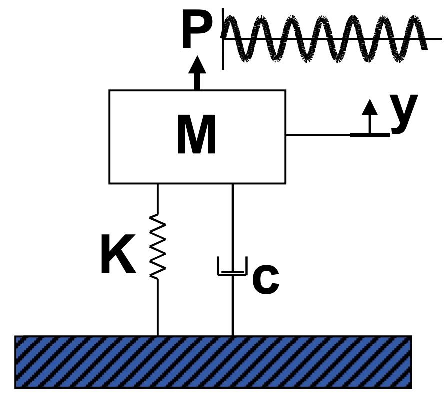
$$=\frac{\left(\frac{f_h}{f_m}\right)^2\cdot\left(1-\left(\frac{f_h}{f_m}\right)^2\right)}{\left(1-\left(\frac{f_h}{f_m}\right)^2\right)^2+\left(2\xi\frac{f_h}{f_m}\right)^2}+i\frac{\left(\frac{f_h}{f_m}\right)^2\cdot 2\xi\frac{f_h}{f_m}}{\left(1-\left(\frac{f_h}{f_m}\right)^2\right)^2+\left(2\xi\frac{f_h}{f_m}\right)^2}$$

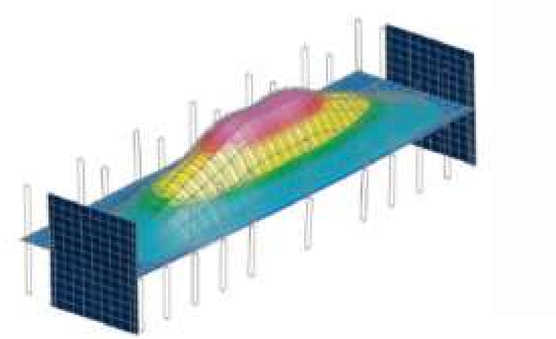
$$F=W\cdot DLF\left(f,harmonic\right)$$

$$Acc=\frac{\rho\cdot F\cdot\varphi_{excition}\cdot\varphi_{reponse}}{m}\cdot FRF_{acc}$$

…Concrete Centre eq 4.4

f_h = harmonic forcing freq.
f_m = modal (structural) freq.





Transient Response

SDOF Impulse – RMS Velocity Response

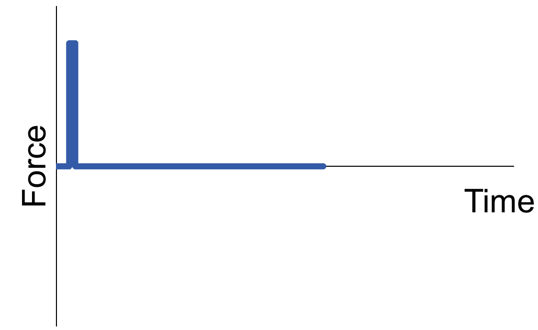
$$m\ddot{y}(t)+c\dot{y}(t)+ky(t)=P(t)=P\cdot e^{i\omega t}$$

$$I_{eff}=54\frac{f_w^{1.43}}{f_n^{1.3}}[\mathrm{Ns}]$$

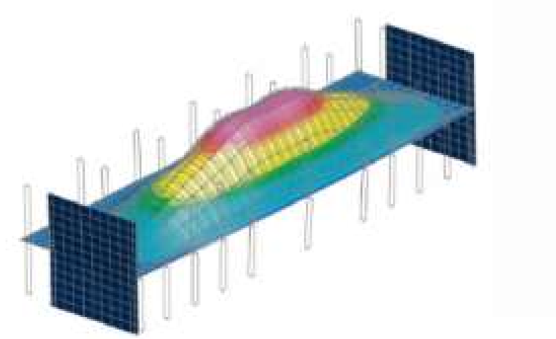
$$Vel_{Peak}=\frac{I_{eff}\cdot\varphi_{excition}\cdot\varphi_{reponse}}{m}$$

$$Vel(t)=Vel_{Peak}\cdot e^{-\omega\xi t}\sin\omega t$$

$$Vel_{RMS}=RMS\left(\sum Vel(t)_m\right)$$




A Design Guide for Footfall Induced Vibration of Structures



…Concrete Centre eq 4.10~4.13

f_w = walking forcing freq.
f_m = modal (structural) freq.

'Bobbing' Resonant Response

MDOF Harmonic Response – crowd interacting with structure

$$M\ddot{y}(t)+C\dot{y}(t)+Ky(t)=P(t)=P\cdot e^{i\omega t}$$

$$\begin{bmatrix} m_s & 0 & 0 \\ 0 & m_a & 0 \\ 0 & 0 & m_p \end{bmatrix}\begin{bmatrix} \ddot{y}_s \\ \ddot{y}_a \\ \ddot{y}_p \end{bmatrix}+\begin{bmatrix} c_s+c_a+c_p & -c_a & -c_p \\ -c_a & c_a & 0 \\ -c_p & 0 & c_p \end{bmatrix}\begin{bmatrix} \dot{y}_s \\ \dot{y}_a \\ \dot{y}_p \end{bmatrix}+\begin{bmatrix} k_s+k_a+k_p & -k_a & -k_p \\ -k_a & k_a & 0 \\ -k_p & 0 & k_p \end{bmatrix}\begin{bmatrix} y_s \\ y_a \\ y_p \end{bmatrix}=\begin{bmatrix} -P \\ P \\ 0 \end{bmatrix}$$

$$FRF_{disp}=\frac{1}{K-M\omega^2+iC\omega}=\begin{bmatrix} k_s+k_a-\omega^2 m_s+i\omega(c_s+c_a) & -k_a-i\omega c_a \\ -k_a-i\omega c_a & k_a-\omega^2 m_a+i\omega c_a \end{bmatrix}^{-1}$$

$$DMF_{disp}=FRF(1,1)-FRF(1,2)=\frac{-\omega^2 m_a}{|FRF|}=\frac{1}{k_a D_a+\left(\frac{f}{f_a}\right)^2-k_s\left(\frac{f_a}{f}\right)^2 D_a D_s}$$

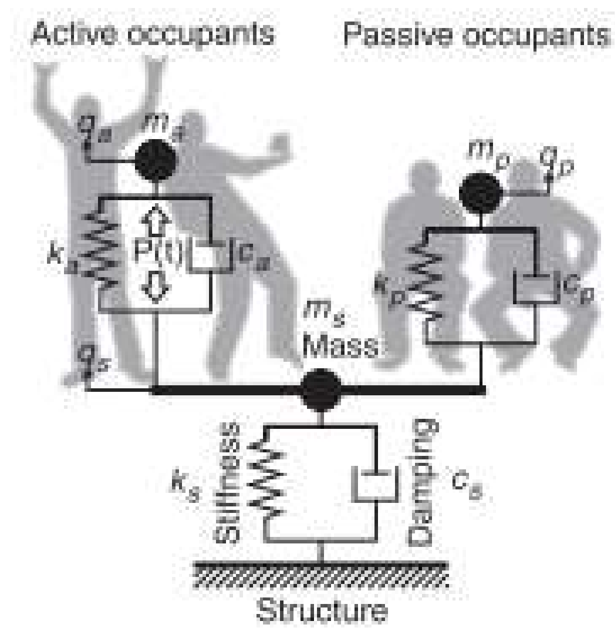
$$D_a=1-\left(\frac{f}{f_a}\right)^2+i2\xi_a\frac{f}{f_a} \qquad D_s=1-\left(\frac{f}{f_s}\right)^2+i2\xi_s\frac{f}{f_s}$$

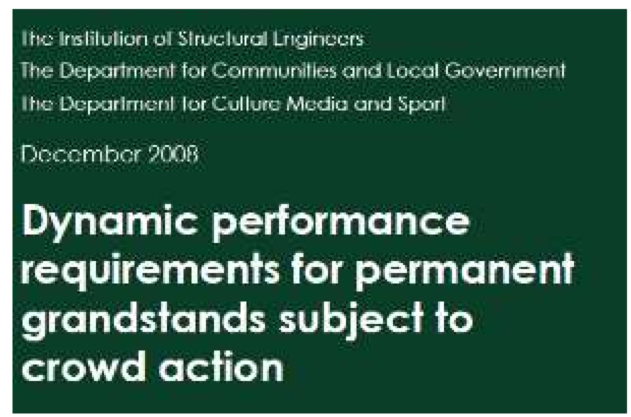
$$P=\rho(f)\cdot W_a\cdot GLF(harmonic, scenario)$$

$$Acc=P\cdot\omega^2 DMF_{disp}$$

…IStructE Route 2

a = active crowd
p = passive crowd
s = structure




The Institution of Structural Engineers
The Department for Communities and Local Government
The Department for Culture Media and Sport

December 2008

Dynamic performance requirements for permanent grandstands subject to crowd action

Programming Implementation

```
foreach node
  // Get response of governing excitation frequency
  R_Node = MAX(R_Node(ƒ))
  foreach excitation frequency
   // Get R Factor
   R_Node(ƒ) = Acc_RMS,Node(ƒ) / Acc_RMS,0(ƒ)
   // Get SRSS acceleration
   Acc_RMS,Node(ƒ) = √(Σ_Mode Σ_Harmonic Acc_RMS(n,m,h,ƒ*h)^2)
   foreach mode
    // Get modal properties
    // modal frequency (ƒ), damping (ξ) & mass (m)
    // displacement at excitation(ϕ_ne) & response(ϕ_nr) nodes
    // <participation factor(ρ_m)>,<static mass (W)>
    ƒ_m,ξ_m,m_m,ϕ_ne,ϕ_nr,W_m,ρ_m = ...
    foreach harmonic
      // Get harmonic properties
      // dynamic amplification factor(DLF), harmonic freq (ƒ)
      DLF_ƒ,h,ƒ_m,h = ...
      // Get RMS acceleration
      Acc_RMS(n,m,h,ƒ*h)=RMS(F(ƒ_m,h,ξ_m,m_m,ϕ_ne,ϕ_nr,DLF_ƒ,h,W_m,ρ_m))
```

…Excel VBA is single threaded so no parallel processing
…Suggest building with parallel libraries in C#, VB, Python, etc

Library floor GSA model

2000 nodes x 20 modes x 4 harmonics x 1.5Hz frequency range = **2.4E7** calculations

Footbridge GSA model

6000 nodes x 40 modes x 2 harmonics x 6Hz frequency range (running) = **2.8E8** calculations

…not practical back in '90s, hence the simplified methods in AISC DG11, AS 5100-2, etc

Some Examples

How can we improve response?

- Increase Damping
- Increase Frequency
    - Increase Stiffness
    - Decrease Mass
- Increase Mass
- Isolate

Further Reading

General Structures (Vertical Resonant or Transient of Pedestrians)

- Concrete Centre CCIP-016 *A design guide for Footfall Induced Vibration of Structures*
- SCI P354 *Design of Floors for Vibration*

Stadia & Concert Hall (Vertical Resonance of Bobbing Crowd)

- IStructE *Dynamic Performance Requirements for Permanent Grandstands Subject to Crowd Action*
- C. Jones, A. Pavic, P. Reynold, R. Harrison *Verification of Equivalent Mass-Spring-Damper Models for Crowd-Structure Vibration Response Prediction*

High Use Footbridges (Lateral Synchronous Lock in and Vertical Crowd)

- P. Dallard *The London Millenium Footbridge*
- BSI PD 6688-2:2011 *Background to National Annex to BS EN 1991-2: Traffic loads on bridges*
- Setra *Footbridges: Assessment of Vibrational Behaviour of Footbridges under Pedestrian Loading*

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